Two-component colour dipole emission in the central region of onium-onium scattering

R. Peschanski

Service de Physique Théorique, CE-Saclay
F-91191 Gif-sur-Yvette Cedex, FRANCE

Abstract

The initial-state radiation of soft colour dipoles produced in the central region of onium-onium scattering via single QCD Pomeron exchange (BFKL) is calculated in the framework of Mueller's dipole approach. The resulting dipole production has a two-component structure. One is constant with energy while the other grows and possesses a power-law tail at appreciably large transverse distances from the collision axis. It may be related to the growth of the gluon distribution at small Bjorken-x.

Résumé

l' émission de dipoles de couleur dans la région centrale d' une collision onium-onium par échange de la singularité BFKL de la QCD est calculée dans la récente approche des dipoles due à Mueller. Une double composante de la section efficace est obtenue par le calcul; L' une est constante en énergie alors que l' autre augmente et possède une distribution en loi de puissance dans la coordonée transverse à l' axe de la réaction. Cette composante est probablement associée à la croissance de la distribution des gluons à petite valeur du paramètre x de Bjorken.

1. Introduction

High energy onium-onium scattering is a simple process which can be used to study the physics of the perturbative QCD Pomeron, the so-called BFKL singularity [1]. Recently a quantitative picture in which a high-Q^2 $q\bar{q}$ (or *onium*) state looks like a collection of colour dipoles of various sizes has been developped by Mueller [2, 3, 4]. The QCD Pomeron elastic amplitude is recovered in this dipole picture provided the onium-onium elastic scattering comes from a dipole in one onium state scattering off a dipole in the other onium state by means of two-gluon exchange [2, 3, 4, 5]. In this paper we elaborate on some consequences of the dipole approach for the initial-state radiation associated with a single Pomeron exchange.

Our starting point is the observation that the onium-onium scattering process is accompanied by a radiation due to colour dipoles which – while present in the initial state – are released during the collision. In this contribution we present an explicit computation of this dipole emission process following Mueller's approach, more specifically that of Ref. [4]. It follows essentially from a recent paper [6] by Andrzej Bialas and myself. As an addendum to the published paper we present an interesting further consequence of this calculation, namely the *two-component* structure of the predicted dipole radiation.

We intend to estimate the emission of colour dipoles from the initial-state. To this end, we first write the formula for the inclusive cross-section for emission of a dipole in the central region from one of the colliding

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onia. It reads:

$$x\ \frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}r^2} = 4\pi\alpha^2 \int_0^\infty \frac{\mathrm{d}l}{l^3}\ \frac{\mathrm{d}x_1}{x_1}\ \frac{\mathrm{d}x_2}{x_2}\left[1-J_0\left(lx_1\right)\right]$$
$$\times\left[1-J_0\left(lx_2\right)\right]\int \mathrm{d}^2b\ \mathrm{d}^2b_1\ \mathrm{d}^2b_2\ \delta^2(b-b_1-b_2)$$
$$\times\ n^{(1)}\left(x_{01},b_1,x_1;\frac{Y}{2}\right) n^{(2)}\left(x_{02},b_2,x_2;r,x;\frac{Y}{2}\right) \quad (1)$$

where $n^{(1)}$ is the *single* dipole density in the onium of size x_{01} and $n^{(2)}$ is the *double* dipole density in the onium of size x_{01}, both dipoles being present in the central region of rapidity $Y/2$.

This formula -which is an extension of Mueller's formula for the total onium-onium cross-section (see Eq.(3) of Ref. [4] - expresses simply the fact that the number of emitted dipoles at rapidity $Y/2$ is just the number of dipoles present at that same rapidity in the initial state *whenever* the interaction took place. The important feature of Eq.(1) is that it contains the double-dipole density $n^{(2)}$ at central rapidity coming from one of the initial onia (since one of the two dipoles is involved in the interaction mechanism). Thus, it is sensitive to correlations between dipoles in the same onium state. As we shall see, this has non-trivial consequences. In fact, already at this stage, one may expect that these correlations should be rather strong because, as shown in Refs. [2, 3, 4], the colour dipoles which contribute to the onium wave function are formed in a cascade process and thus cannot be independent. Furthermore, since this cascade is scale-invariant one also expects scale invariance in the dipole-dipole correlations to be valid, which -in turn- is likely to be a reflection of the inner conformal invariance, known to be rooted in the formalism of the BFKL Pomeron [7].

2. The dipole emission cross-section

2.1. *The Mellin Transform of the cross-section*

The appropriate expression for $n^{(1)}$, was given in Ref. [4]:

$$n^{(1)}\left(x_{01},b,x;Y/2\right) \equiv \frac{x_{01}}{4x\left|b\right|^2}\ \ln\frac{\left|b\right|^2}{x_{01}x}\ \exp\left(-a\ \ln^2\frac{\left|b\right|^2}{x_{01}x}\right)$$
$$\times\exp\left[\left(\alpha_p-1\right)Y/2\right]\left(\frac{4a}{\pi}\right)^{3/2}\quad , \quad (2)$$

where

$$a \equiv a(Y) = \left[7\alpha\ N_c\zeta(3)Y/\pi\right]^{-1} \approx \left[3\left(\alpha_p-1\right)Y\right]^{-1}. \quad (3)$$

The needed explicit expression for $n^{(2)}$ has been derived in our paper [6] extending the formalism of Ref. [4]. It makes use of an inverse Mellin transform, that is:

$$n^{(2)}\left(x_{02},b_2,x_2;r,x;Y\right) = \int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}\gamma}{2i\pi}\ x_{02}^{\gamma}\ \times$$
$$\times\ \tilde{n}^{(2)}\left(\gamma,b_2,x_2;r,x;Y\right) \quad (4)$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}r^2}(x_{02}) = \int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}\gamma}{2i\pi}\ x_{02}^{\gamma}\ \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x\mathrm{d}r^2}(\gamma)$$

with the real constant $c<2$ for convergence condition.

Using the hypergeometric function

$${}_2F_1\left[y^2\right] \equiv\ {}_2F_1\left(\frac{\gamma}{2},\frac{\gamma}{2};1;y^2\right),$$

one obtains as a final result:

$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x\mathrm{d}r^2} = \sigma_{\mathrm{tot}}\ 2\sqrt{2}\ \frac{\alpha N_c}{\pi}\left(\frac{2a}{\pi}\right)^{3/2}\int_0^1 \mathrm{d}y\ {}_2F_1\left[y^2\right]$$
$$\times\quad \frac{r^{-\gamma}}{x}\ \frac{\exp\left[\left(\frac{\alpha N_c}{\pi}\chi(\gamma)-(\alpha_p-1)\right)\ Y/2\right]}{2\ \left(\alpha_p-1\right)\ -\ \frac{\alpha N_c}{\pi}\chi(\gamma)} \quad (5)$$
$$\times\quad \frac{\ln\left(\frac{r}{x}\right)\ \exp\left[-a\ln^2\left(\frac{r}{x}\right)\right]}{2+2a\ln\left(\frac{r}{x}\right)-\ \gamma}.$$

where, by definition,

$$\chi\left(\gamma\right) \equiv 2\psi(1) - \psi\left(1-\gamma/2\right) - \psi\left(\gamma/2\right), \quad (6)$$

$\psi(\gamma) \equiv \frac{\mathrm{d}}{\mathrm{d}\gamma}\ln\,\Gamma,\ \chi(1)\ \frac{\alpha N_c}{\pi} \equiv \alpha_p - 1 =\ 4\ln 2\ \frac{\alpha N_c}{\pi}.$

2.2. *The two components of the cross-section*

The inverse Mellin transform of (5) has to be determined in order to obtain the expression for the emission cross-section $\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}r^2}$. In performing it, one has to take into account the different singularities in the $\gamma-$variable present in expression (5). There are of two kinds, namely the poles in the denominators, and the well-known saddle-point in the function $\chi(\gamma)$ at $\gamma=1$. These contributions lead to a two-component structure of the dipole radiation in the central region.

2.2.1. Component I : the saddle-point The saddle-point contribution comes from the second row of the expression (5). One gets:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}r^2}\right)_I\ =\ \sigma_{\mathrm{tot}}\ \frac{1}{xr^2}\ \left(\frac{r}{x}\right)\varphi_I\left(\frac{x}{r},\frac{x_{10}}{r}\right)\quad , \quad (7)$$

where [4] $\sigma_{\mathrm{tot}} = 2\pi\ x_{10}x_{20}\ \alpha^2\ \mathrm{e}^{(\alpha_p-1)\ Y}\ \left(\frac{2a}{\pi}\right)^{1/2},$

$$\varphi_I \approx \mathcal{C}st.\ \left(\frac{2a}{\pi}\right)^2\ \ln\frac{r}{x}\ \mathrm{e}^{-a\ \left(\ln^2\frac{r}{x}+\ \ln^2\frac{r}{x_{10}}\right)}\ . \quad (8)$$

2.2.2. Component II : the pole One has to take into account the poles present in the denominators of expression (5). In fact the one which is less but closest to the $2 \pm i\infty$ line in the complex γ-plane will dominate. One obtains:

$$\frac{\alpha N_c}{\pi}\,\chi\,(\gamma^*) = 2(\alpha_p\ -\ 1) \equiv\ \frac{2\ \alpha N_c}{\pi}\ \chi(1),$$

Looking for a solution of the form $\gamma^* = 2 - \gamma_M$ and using the explicit form of the kernel $\chi(\gamma)$ leads to a value $\gamma_M \approx .37$. Notice that the other denominator in (5) corresponds to a pole outside the integration contour (at $\gamma > 2$).

One finally gets:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}r^2}\right)_{II} = \sigma_{\mathrm{tot}}\ \frac{1}{xr^2}\ \mathrm{e}^{\left((\alpha_p-1)\frac{Y}{2}\right)}\ \frac{x_{10}}{x}\left(\frac{r}{x_{10}}\right)^{\gamma_M}\ \varphi_{II}, \tag{9}$$

$$\varphi_{II} \approx \mathcal{C}st.\left(\frac{2a}{\pi}\right)^{\frac{3}{2}}\ \ln\ \frac{r^2}{xx_{20}}\ \mathrm{e}^{-a\ln^2\left(\frac{r^2}{xx_{20}}\right)}. \tag{10}$$

3. Conclusion

The interpretation of the two components (I and II) of the inclusive cross-section in terms of conventional gluon diagrams deserves more study and their specific properties bring some unexpected features. Let us briefly outline some intersting aspects.

The first component is approximately independant of the energy apart logarithmic factors. It is tempting to associate it with a similar contribution found in the derivation of the Pomeron-Pomeron contribution for gluons in the BFKL formalism [8]. By contrast, the second component has a rather strong energy dependence. In fact, it is related to the large rapidity difference between the initial dipole x_{20} and the one emitted in the very central region near the rapidity $Y/2$. It means a very small value of $x_{BJ} \approx e^{-Y/2}$ and consequently, the inclusive cross-section is sensitive to the singular behaviour of the gluon distribution at small x_{BJ} related to the BFKL singularity. The properties of this component have been examined in detail in our paper [6], showing in particular the interesting feature of a power-law tail in the transverse distance with respect to the collision axis.

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